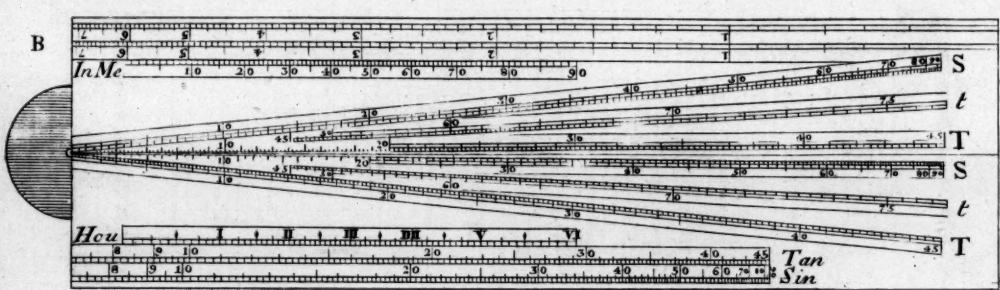
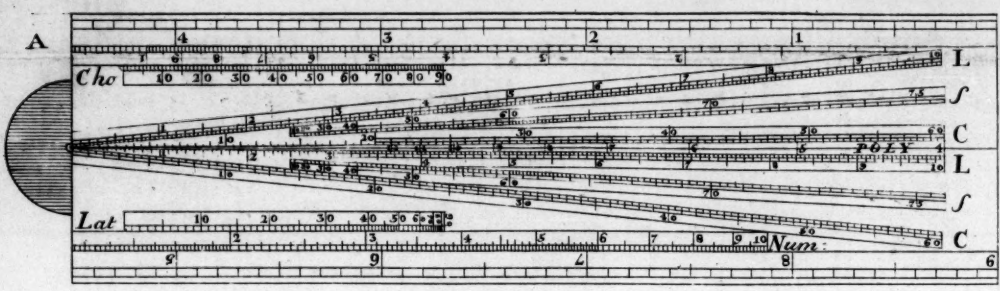
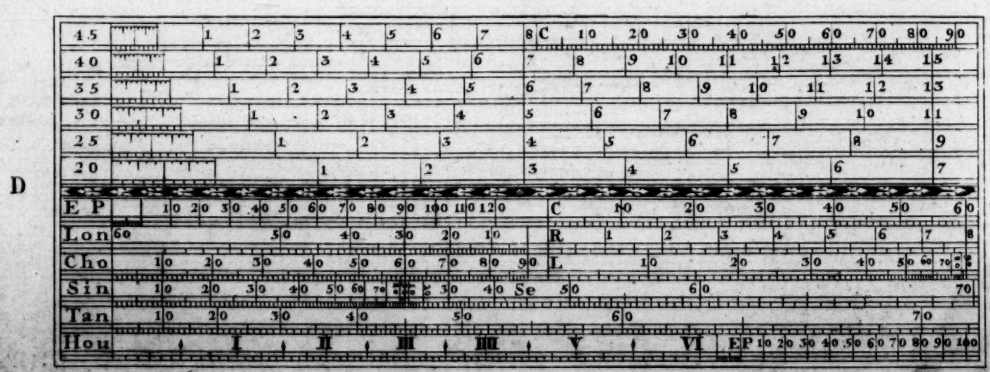
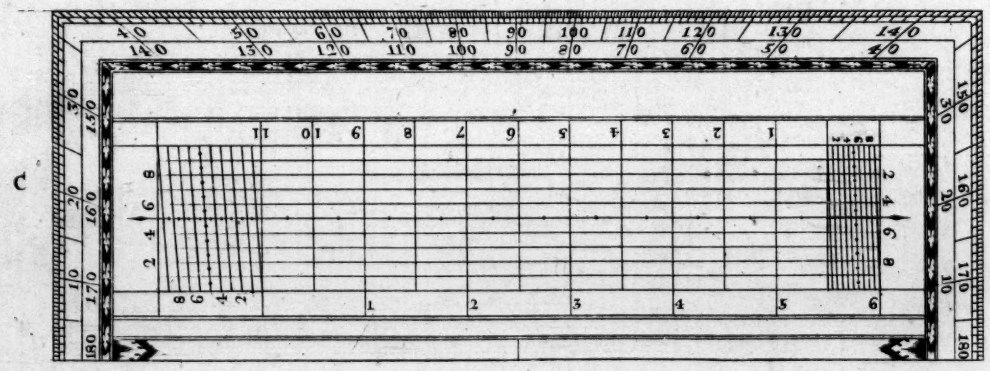


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THE
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CONTAINING THE
CONSTRUCTION of the several LINES
LAID DOWN ON THE
PLAIN-SCALE, and SECTOR;
WITH THEIR
APPLICATION,
IN VARIETY OF
MATHEMATICAL PROBLEMS.

BY
WILLIAM WEBSTER.

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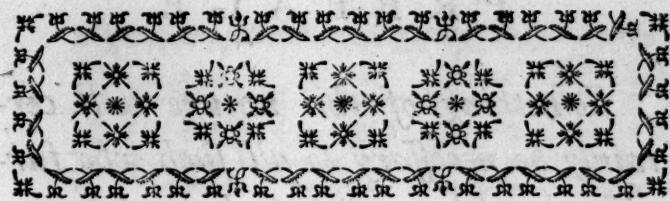


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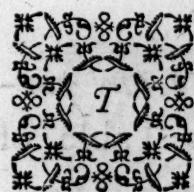
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P R E F A C E.



THE following sheets were first printed as an Appendix to A compendious course of Practicall Mathematicks, which I published some time ago, and contain, as is set forth in the title, The description and use of a complete sett or case of pocket-instruments. Ever since that publication, I have been much urged to print the said Appendix separately, as a thing that would be more generally useful: I have at last complied, and wish it may prove as acceptable to the Publick, as some private friends would persuade me it will be.

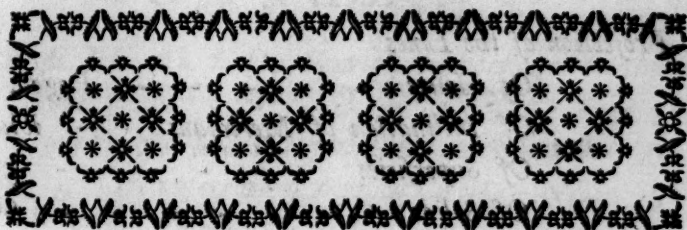
P R E F A C E.

Something of this nature will certainly be necessary to all such who furnish themselves with such an Apparatus; and I am apt to believe, that what I have herein delivered will be sufficient for the purpose.

For the ready turning to any particular, I have prefixed a Table of Contents, and therefore think it needless to add any thing farther by way of Preface to a Tract so short, yet, I hope, clear and intelligible.



CON-



C O N T E N T S.

	<i>General Description</i>	Page 1
	<i>Of the Compasses</i>	3
	<i>Of the longer Drawing-pen with the</i>	
	<i>Protracting-pin</i>	4
	<i>Of the parallel Ruler</i>	ibid.
	<i>Of the Protractor</i>	5
<i>Of Scales or Lines of equal Parts</i>		
	<i>Simply divided</i>	7
	<i>Diagonally divided</i>	8
	<i>The Construction and Use of the Plain Scale</i>	10
<i>Projection of the Lines</i>		
	<i>Of Tangents</i>	ibid.
	<i>Of Semi-tangents</i>	11
	<i>Of Secants</i>	ibid.
	<i>Of Sines</i>	ibid.
	<i>Of Chords</i>	ibid.
	<i>Of Rhumbs</i>	12
	<i>Of Longitude</i>	ibid.
	<i>Of Latitude</i>	13
2		Pro-

C O N T E N T S.

Projection of the Lines

Of Hours - - - - -	Page 14
Of Inclination of Meridians - - -	ibid.
Of Superficies - - - - -	59
Of Solids - - - - -	60

The Uses of the Lines

Of Tangents, Semi-tangents and Secants	14
Of Sines - - - - -	ibid.
Of Chords - - - - -	15
Of Rhumbs - - - - -	16
Of Longitude - - - - -	ibid.
Of Latitude and of Hours - - -	17
Of Superficies - - - - -	59
Of Solids - - - - -	61

To construct an horizontal Dial by the Lines of Latitude and Hours, for the Latitude of $51^{\circ} 32'$ 17

Description of the Sector - - - - - 19

Of the Foundation and general Use of the Sector 22

Of the particular Use of the Lines of Lines in several Problems, viz.

Prob. I. To divide a given Line into any Number of equal Parts - - - - - 26

II. To divide a Line in the same Manner as another is divided - - - - - 27

III. To find the Proportion between two or more given Lines - - - - - 28

IV. To increase or diminish a Line in any given Proportion - - - - - 29

V. To find a Third, proportional to two given Lines - - - - - 30

Prob.

CONTENTS.

<i>Prob. VI. To find a Fourth, proportional to three Lines given</i>	- - - - -	Page 31
<i>VII. To find a Right Line nearly equal to the Circumference of a given Circle</i>	- - -	32
<i>VIII. To open the Sector, so that the two Lines of Lines, &c. may stand at right Angles with each other</i>	- - - - -	ibid.
<i>IX. To open the Lines upon the Sector, or the Edges of the Instrument itself, to any proposed Angle</i>	- - - - -	33
<i>X. The Sector being opened, to find the Quantity of the Angle</i>	- - - - -	34
<i>Of the Use of the Lines of Polygons</i>	- - -	ibid.
<i>Of the Lines of artificial Numbers, Sines and Tangents</i>	- - - - -	36

The Manner of working Proportions, exemplified in the Resolution of the several Cases of plain Trigonometry, both right-angled and oblique-angled.

The Seven Cases of right-angled plain Triangles.

<i>Case I. The Angles and one of the Legs being given, to find the other Leg</i>	- - - - -	42
<i>II. The Angles and one of the Legs being given, to find the Hypothenuſe</i>	- - - - -	44
<i>III. The Hypothenuſe and Angles given, to find either of the Legs</i>	- - - - -	45
<i>IV. The Legs being given, to find the Angles</i>		46
<i>V. The Legs given, to find the Hypothenuſe</i>		47
<i>VI. The Hypothenuſe and one of the Legs given, to find the Angles</i>	- - - - -	48
		Case

CONTENTS.

Case VII. The Hypotenuse and one of the Legs being given, to find the other Leg - - - Page 49

The Six Cases of oblique-angled plain Triangles.

Case I. Two Sides, and an Angle opposite to one of them, being given, to find the other opposite Angle

50

II. The Angles and one Side being given, to find either of the other Sides - - - 52

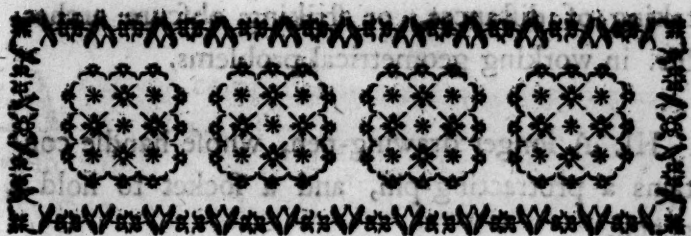
III. Two Sides, and an Angle opposite to one of them, being given, to find the other Side 53

IV. V. Two Sides and the contained Angle being given, to find the other two opposite Angles, and the other Side - - - - - 54

VI. The three Sides being given, to find the Angles - - - - - 56



THE



THE
DESCRIPTION and Use
Of a Complete
SETT or CASE
of
POCKET-INSTRUMENTS.



Complete sett of pocket-instruments (though sometimes made according to particular direction) generally consists of the following particulars, *viz.*

I. A large pair of compasses, with one point made to take out, in order to receive in its place, either a drawing-pen, a port-crayon, or a dotting-wheel; according as you would describe your circles or arches, with ink, black-lead, &c. or only with dots.

B

II. A

II. A smaller pair of compasses, for the exact taking of distances, or striking obscure arches, &c. in working geometrical problems.

III. A longer drawing-pen, whose handle contains a protracting-pin, and a socket to hold a pencil.

IV. A parallel ruler.

V. A protractor, either semicircular, or (as they are more commonly fitted up for cases) in form of a parallelogram or long square, for the conveniency of laying down on the back side thereof, several lines of equal parts, both simply and diagonally divided, with two or three lines of chords of different radius's; and sometimes all the several lines of the common plain scale are laid down thereon.

VI. A sector,

These cases of instruments are made of several sizes; but most commonly either of the length of four inches, four inches and a half, or six inches; which last size, though somewhat heavier for the pocket, are much more preferable in use, as the divisions upon the sector are more sensible, and consequently their application in practice more exact.

Of the Compasses.

A Pair of compasses is a thing so well known, that it would be little less than ridiculous to waste words in its description; but a few hints of direction, in judging of their goodness, may probably be of service. Observe therefore, 1. That the motion of the head be equal; that is, that in opening or shutting you find no sudden jerks or leapings. 2. That the joints are well fitted, so that there be no shake in the application of any of the points. 3. That they be well filed and polished. And, lastly, That the steel points are close joined, equal, and well tempered.

The use of a pair of compasses is as well known as their form; I have therefore little to say under this head, but that it is an instrument well adapted to the taking of distances, and describing circles, or arches, from a given center; which, as before was intimated, may be done either obscurely, with the steel point; in ink, with the drawing pen point; in black lead, &c. with the crayon point; or by dots only, with the dotting wheel point; as the occasion requires. Note, also, either of these points may be used to draw a strait line along the edge of a ruler.

4 *The Description and Use of*

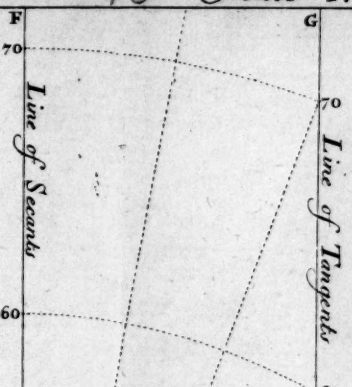
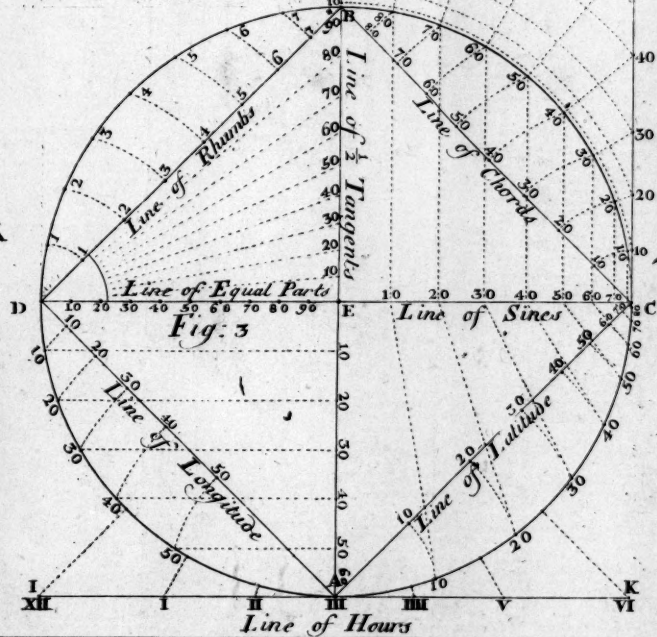
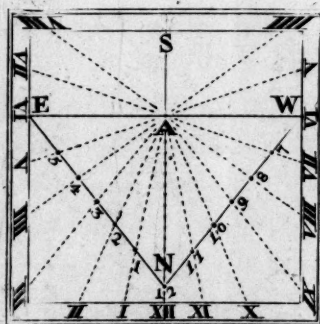
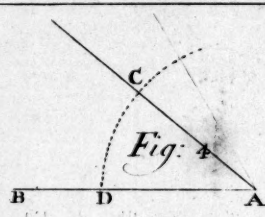
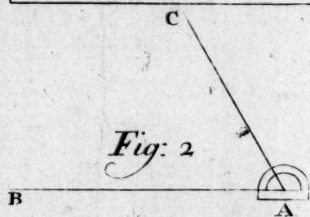
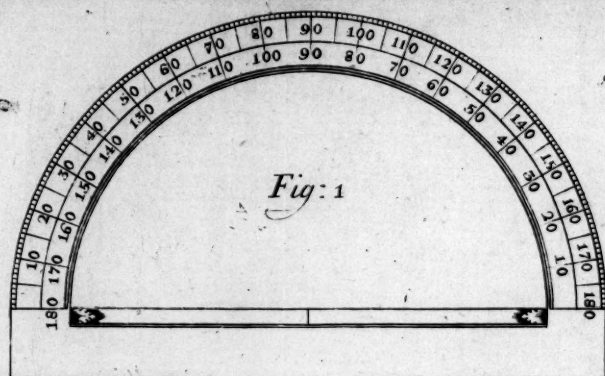
Of the longer drawing Pen, with the protracting Pin.

THIS instrument is composed of two steel blades, fixed into a small pillar of brass, silver, &c. one of which, by means of a joint, may be made to open for the conveniency of cleaning; and by help of a little screw, they are easily set to the proper thickness of the line you would draw; the ink being inserted into the cavity betwixt the blades, with a pen; so that if the blades are well fitted and polished, the line they draw will be exceeding smooth and even; which cannot be so certainly done with a common pen. The use therefore of this part of the instrument is only to draw strait lines; which generally should be drawn very fine. The other part, which is called a *protracting pin*, is only a short piece of fine steel wire, fix'd in one end of the other part of the handle: and its use is to set off angles from the protractor, as shall be hereafter shewn in describing that instrument.



Of the parallel Ruler.

THIS instrument consists only of two equal rulers, so connected to each other by moveable cross-pieces fix'd parallel to each other, and lying asslant, when the two rulers are closed, or crossing each other with a joint in the middle, that



to whatever degree the rulers are opened, they are still kept in a true parallel position to each other; by which means, one part being set true upon any line, another line may be drawn by the other part of the ruler exactly parallel to that whereon the first is placed; which expedient is of great use in delineating most sorts of draughts, particularly in plans of architecture, both military and civil; and in reducing a multangular figure to a triangle, for the more accurate and ready finding the content.



Of the Protractor.

THE protractor is commonly a semi-circle, on whose chord or diameter *Fig. 1.* is mark'd its centre, and whose arch is divided into 180 equal parts, called degrees; being the half of 360, which every whole circle is supposed to be divided into. (*See fig. 1. plate I.*) But for cases of instruments, it is frequently made in form of a parallelogram, upon three sides of which the same divisions are laid down from a centre, mark'd on the fourth (as in the plate fronting the title, in the figure mark'd C). Every 10 of these degrees are numbered, and every fifth distinguished with a longer mark than the others, for the greater ease in reckoning. They are usually numbered both ways, *viz.* from right to left, and from left to right, for the more readily laying down an angle

6 *The Description and Use of*

angle from either end of a given line. The side to be applied to the paper is made flat, and that whereon the degrees are mark'd is fil'd so sloping as to render the edges thin for the greater exactness in setting off, or measuring an angle; which are all the uses of this instrument as a protractor, and perform'd in the following manner:

1. If it was required to lay down an angle of 60 degrees, from the point A of the line AB; lay the diameter of the protractor on the line AB, so that the center of the instrument may fall upon the point A; then with your protracting pin make a mark exactly against the number 60 upon the limb of your instrument; so shall a line drawn through the said mark to the point A, form an angle of 60 degrees.

2. If it be required to find the content of an angle given, as in the foregoing figure the angle BAC; lay the diameter of the protractor on the line AB, so that the centre may fall exactly upon the point A; then shall the degree on the limb of the instrument cut by the line AC, shew the content of the angle as before, 60 degrees.

On the reverse of a protractor, when made in form of a parallelogram, (*Vide* figure mark'd D in the plate fronting the title) are usually laid down several scales, or lines of equal parts of different proportions, that is, of 45, 40, 35, 30,

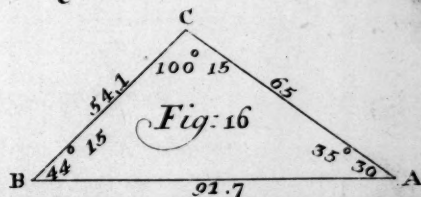
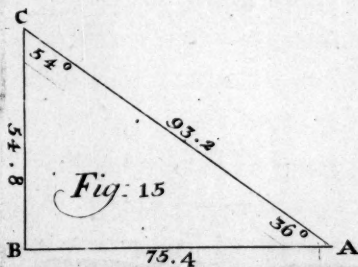
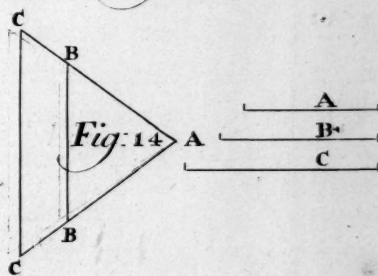
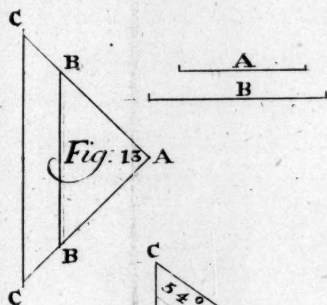
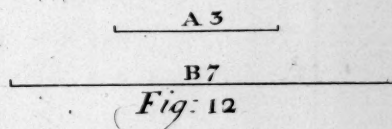
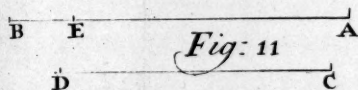
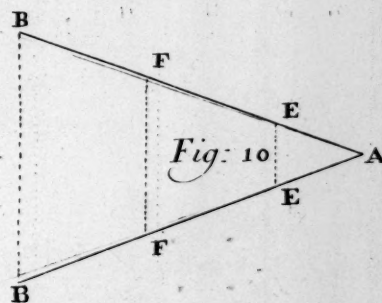
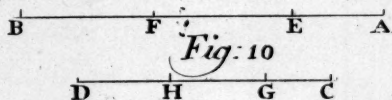
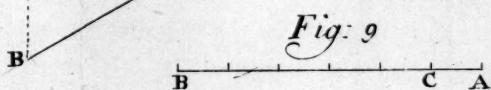
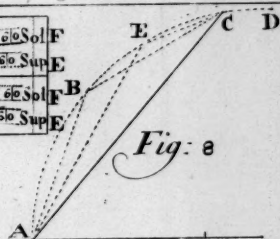
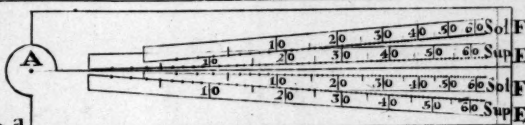
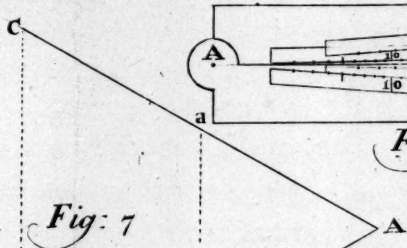
25, and 20 parts to an inch, distinguish'd by the said numbers prefix'd, those of 20 and 40 being commonly diagonally divided; likewise lines of chords of different radius's; and if the diagonal scales be laid down within the vacant space on the face of the protractor, (as in fig. C.) there may be room on the reverse (as in fig. D.) for all the lines usually placed on the instrument, called the *plain scale*, as well as the before-mentioned lines of equal parts, &c. In giving therefore the construction and use of the plain scale, I shall sufficiently describe all the lines ever laid down on the reverse of a protractor, and shew their several uses and application in practice. But first I shall shew the use of these several scales or lines of equal parts, both as they are simply and diagonally divided.

Of the Use of Scales, or Lines of equal Parts.

1. *Simply divided* (Vide fig. D.) A line of equal parts is no more than a scale of measure in miniature, for laying down by way of plan, lengths, taken by any known measures, as chains, yards, feet, &c. each part on the scale answering accordingly to one chain, one yard, or one foot, &c. The plan therefore will be the greater or less; as the scale used contains more or less parts in the space of an inch. Note, the first division on every scale is subdivided into 10 equal parts, which

which smaller divisions are the parts to be understood, when we say it is a scale of 20 or 30, &c. If therefore these lesser divisions be taken as units, and each represent one league, one mile, one chain, one yard, or one foot, &c. the larger divisions will consequently be tens, tho' number'd with single figures; so that to set off 36 leagues, miles, chains, yards or feet, &c. extend the compasses from 3 of the larger divisions, to 6 reckon'd backwards upon the smaller, and that extent laid down on your plan shall accordingly represent 36 leagues, miles, chains, yards or feet, actually measured in the thing represented. To adapt these scales to foot measure, you will likewise find the first division of each duodecimally divided, above the decimal division, the halves and quarters of which are distinguished by longer marks than the rest, which are generally only dots; so that to lay down any number of feet and inches, as suppose 8 F. 9 I. extend the compasses from 8 of the larger divisions, to 9 of the upper smaller divisions; and the said extent laid down on your plan, shall represent by whatever scale it is taken from, 8 F. 9 I. in length actually measured. This may suffice for those scales which are only simply divided.

2. *Diagonally divided* (*Vide fig. C.*) For those which are more accurately divided by diagonal lines, as one of the larger divisions are thereby exactly divided into 100 equal parts, if the scale contains





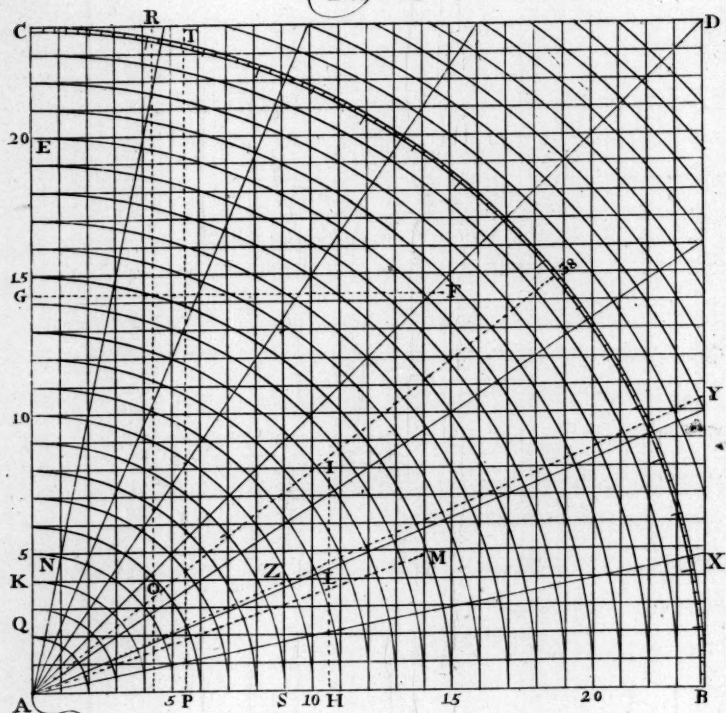
expressible by the same extent of the compasses; viz. setting one foot in number 5 of the larger divisions, extend the other along the sixth parallel, to the seventh diagonal; for if the 5 larger divisions are taken for 5 hundred, 7 of the first subdivisions will be 70, which upon the sixth parallel, taking in 6 of the second subdivisions, or 6 units, makes the whole number 576. Or, if the five larger divisions be taken for 5 tens or 50, seven of the first subdivisions will be 7 units, and the 6 second subdivisions, included upon the sixth parallel, will be six tenths of an unit. Lastly, if the 5 larger divisions be only esteem'd as five units, then will the 7 first subdivisions be seven tenths, and the 6 second subdivisions six hundredth parts of an unit. And this, I think, may be sufficient to shew the use both of the simple and diagonal scales of equal parts: I now proceed to

The Construction and Use of the Plain Scale.

Having described the circle ABCD, and divided it into four quadrants, by the diameters AB and CD, crossing each other at right angles;

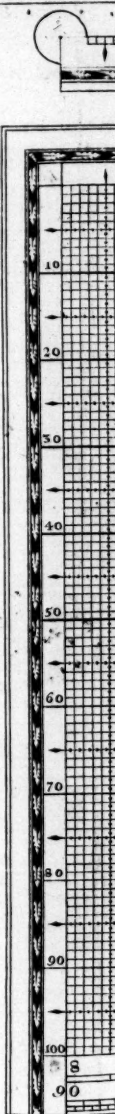
1. To project the line of *Tangents*: From the end C of the diameter CD, erect the perpendicular CG; then dividing the arch CB into nine equal parts, from the centre E, through the several divisions of the arch, draw lines, till they cut

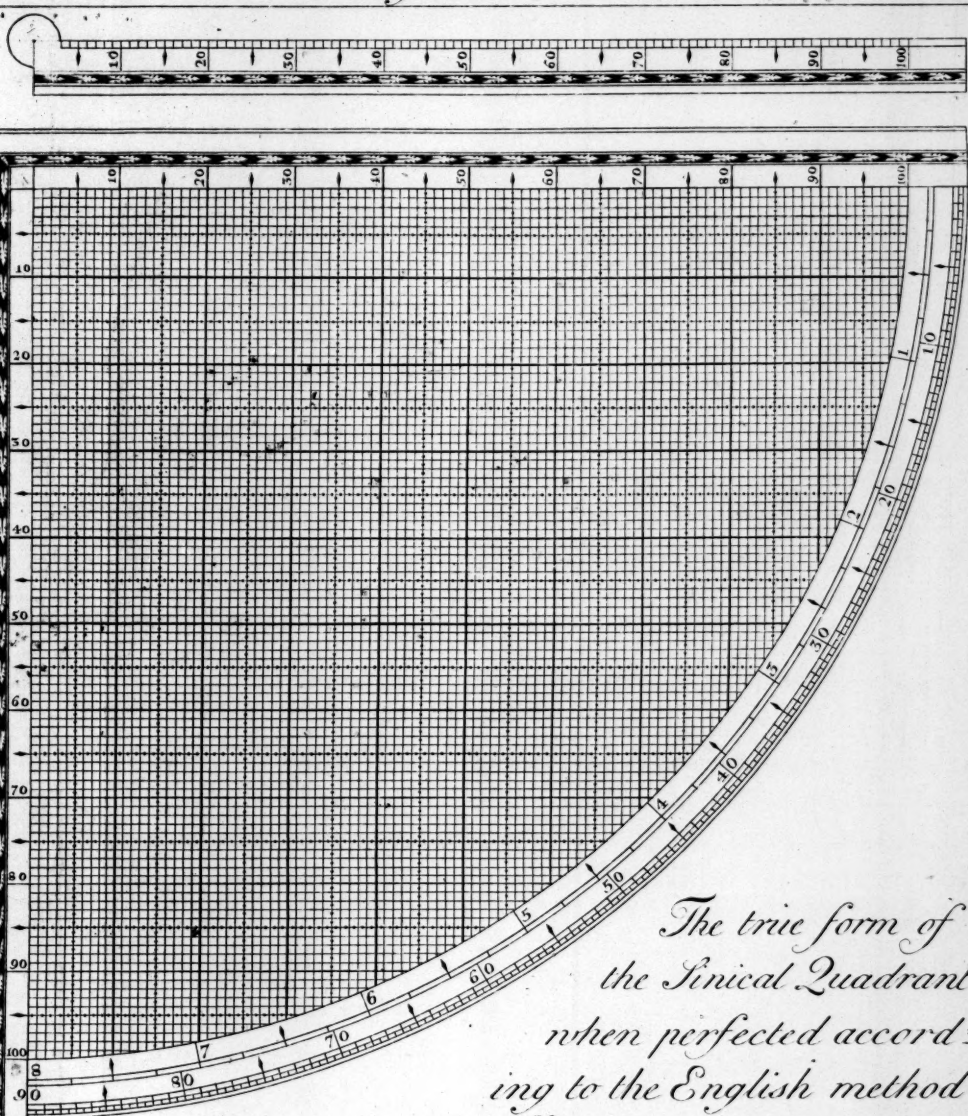
Fig: 25



E

F





The true form of
the Sinical Quadrant
when perfected accord-
ing to the English method
of Projection



cut the perpendicular CG , which will thereby become a line of tangents.

2. For the *Semi-tangents*: Lines drawn from the point D , through the same divisions upon the arch CB , will divide the radius BE , into a line of semi-tangents. For the semi-tangent of any arch is but the tangent of half such arch; and by the 20th of the 3d of *Euclid*, it is proved, that an angle at the centre is the double of one at the circumference.

3. For the *Secants*: Since the lines drawn from the centre through the several divisions of the quadrant CB , to form the tangents, are the secants of those respective arches, transfer those lengths to the line EB continued to F , and the line EF will be a line of secants.

4. For the *Sines*: From the several divisions of the arch CB , let fall perpendiculars upon the radius CE ; so shall the radius CE be divided into a line of sines, which is to be numbered from E to C for the right sines, and from C to E for the versed sines. Note, the versed sines may be continued to 180 degrees, if the same divisions be transferred on the other side of the centre E .

5. For the *Chords*: The arch CB being divided into 9 equal parts, in the points 10, 20,

C 2

30,

30, &c. if therefore you imagine lines drawn from C, to the said several divisions, they will be the chords of their respective arches. Setting therefore one foot of your compasses in the point C, and transferring the several lengths C 10, C 20, C 30, &c. to the line CB, and the line CB shall thereby be made a line of chords.

Note, *These several lines (which in the figure are drawn but to every 10 degrees) might be constructed to every single degree, if the circle were made large enough to admit of 90 distinct divisions in the arch of its quadrant,*

6. A line of *Rhumbs* is thus constructed: Divide the arch DB into 8 equal parts, in the points 1, 2, 3, 4, &c. then setting one foot of the compasses in D, transfer the several distances D 1, D 2, D 3, &c. from the arch to the line DB, by which means you will form a line of rhumbs, each of which will answer to an angle of $11^{\circ} 15'$.

7. For the line of *Longitude*: Divide the radius EA into 60 equal parts, (because 60 miles make a degree of longitude under the equator) marking every ten with their proper number; from these divisions let fall perpendiculars upon the arch AD; then having drawn the line AD, with one foot of the compasses in A, transfer the several distances, where the perpendiculars cut the arch, to the line AD; which shall thereby be made a line of longitude.

Now,

Now, if the several lines, with their respective divisions thus projected in fig. 3. be transferr'd to the scale mark'd D, in the plate fronting the title, where they are distinguish'd by their proper names; and thereunto be added lines of equal parts, as before describ'd, the instrument called the *Plain Scale* will thereby be form'd.

But as upon the reverse of some protractors, besides the several lines already explain'd, there are two other lines laid down, *viz.* the line of latitude, and the line of hours; I shall likewise shew the manner how they are projected.

Projection of the Lines of Latitude, and of Hours.

1. To project the line of *Latitude*: In the same diagram (*viz.* fig. 3.) the radius CE being already divided into a line of fines, laying a ruler from the point B, through every of the said divisions, mark the points cut on the opposite arch AC, with the numbers 10, 20, 30, &c. Then having drawn the right line AC, with one foot of your compasses in A, transfer the several intersections of the arch to the said right line; which will thereby become a line of latitude, and may accordingly be laid down on your scale.

2. To project the *Hour-line*: Draw the tangent IK, equal and parallel to the diameter CD; and divide half the arch of each quadrant AC and AD, from the point A, into three equal parts, which will be 15 degrees each part, for the degrees of every hour from 12 to 6; each of which parts are to be again subdivided into halves and quarters, &c. Then drawing lines from the centre of the circle, through each of these divisions and subdivisions, till they cut the tangent line IK, the said tangent line will thereby be divided into a line of hours.

3. To these may be still added another line, applicable to the same use, viz. the line of *Inclination of Meridians*, which is projected in the same manner as the hour-line; only divided into degrees, instead of time, every 15 degrees being equal to an hour.

The uses of the several lines thus laid down on the scale D, from the foregoing projection, are as follow:

1. The *Tangents*, *Semi-tangents* and *Secants* serve to find the centres and poles of projected circles, in the stereographical projection of the sphere, &c.

2. The *Sines* serve for the orthographical projection of the sphere, &c.

3. The

3. The line of *Chords* serves, either to set off an angle from a given point in any right line, or to measure the quantity of an angle already laid down. The first is done, by opening your compasses to the extent of 60 degrees upon the line of chords, (which is always equal to the radius of the circle of projection) and setting one foot in the angular point, with that extent describing an arch; then taking the quantity of the angle required, between the feet of your compasses, from the same chord-line, set it off from the given line upon the arch described: so shall a right line drawn from the given point, through the point marked on the arch, form the angle required. Thus to lay down from the point A of the line AB, an angle of 40 degrees, taking the extent of 60 degrees between the feet of your compasses from the line of chords, strike the arch CD; then setting off upon the said arch from D to C, the extent of 40 degrees taken from the said line of chords, from the point A draw the line AC, to pass through the point C; so shall the lines AB and AC form an angle of 40 degrees. The latter is done by describing, as before, an arch with the extent of 60 degrees, and then taking the extent betwixt the lines which form the angle upon the said arch; that extent, measured upon the same line of chords, will shew the quantity of the angle.

Fig. 4.

4. The

4. The line of *Rhumbs* serves with more readiness than the line of chords, to lay down or measure the angle of a ship's course in navigation. For example, suppose a ship's course being N.N.E. it is required to lay down the angle.

Fig. 5. Draw the line *A B* for the meridian, then with 60 degrees of the chords describe from the point *A*, the arch *BC*; and as N.N.E. is the second rhumb from the north, open your compasses to the second rhumb, and lay off the extent upon the arch *BC*; so shall a right line from the point *A* through the point *C*, give the angle of the course *BAC*. Or if it were required to find the quantity of the angle of a ship's course *BAC*; having described the arch *BC*, with 60 degrees of the chords, take the distance of the lines upon the said arch between the feet of your compasses; which measured upon the line of rhumbs, will shew the quantity of the angle.

5. The line of *Longitude* being laid down on the scale contiguous with a line of chords of the same radius, numbered the contrary way, shews, by inspection, how many miles there are in a degree of longitude, in each parallel of latitude, accounting the latitude upon the line of chords, and the miles of longitude upon the line of longitude. Thus in the latitude of 30 degrees, it appears, that 52 miles make a degree of longitude; in the latitude of 60 degrees, 30 miles make a degree; and

and in the latitude of 80 degrees, 10 miles make a degree, &c.

6. The two lines of *Latitudes* and *Hours* are used conjointly, and serve very readily to mark the hour-lines in the construction of dials, on any kind of upright plains.

As an instance of which, I shall give one example of an horizontal dial, for the latitude of *London*, viz. $51^{\circ} 32'$.

To construct an horizontal Dial, by the Lines of Latitude and Hours; for the Latitude of $51^{\circ} 32'$.

First, draw the line E W of a convenient length for the six o'clock line, and at right angles thereto, the line N S for *Fig. 6.* the meridian, or 12 o'clock line, of a breadth equal to the thickness of the style; the interfection of which lines, viz. the point A, will be the centre of your dial: Then taking $51^{\circ} 32'$ (the latitude of the place) from the line of latitude, with one foot of your compasses in A, set off that extent on both sides, from A to E, and from A to W; then with the whole length of the line of hours between your compasses, set one foot in E, and cross the meridian line with the other; do the same on the other side from W, and draw

D the

the lines NE and NW; then taking between your compasses from the line of hours, the distance from the beginning thereof to I, lay down that distance both ways from N to 1, and from N to 11, likewise from E to 5, and from W to 7; set off also the distance from the beginning of the line to II on the line of hours, both from N to 2 and 10, and from E to 4, and W to 8; and lastly, lay down the distance from the beginning of the line of hours to III, from N to 9, and from N to 3; which said distance will likewise reach from E to 3, and from W to 9. Thus having divided your trigon into its proper distances, lines drawn from the centre A of the dial, through these several divisions, will mark the respective hour-points on the limb of the dial, whether its figure be square or round; and for the hours before 6 in the morning, and after 6 at night, they are easily found, by continuing the lines of the opposite hours through the centre.

Note, The style, which is generally a thin plate of brass, must make an angle with the plain, equal to the latitude of the place, the angular point being fixed just at the centre A, and the style standing upon the meridian line, at right angles with the plain of the dial.

Of the SECTOR.

THE *Sector* is a mathematical instrument, taking its name from a geometrical figure, which *Euclid* thus defines : A sector is a part of a circle, bounded by two radii or semidiameters, and part of the circumference, *viz.* the included arch. As an instrument, it consists of two equal legs, moveable about a joint, like the common carpenters rule ; which being each of the length of six inches, (tho' they may be made of any other determinate length, as in the figures mark'd A and B in the plate fronting the title, they are of 4 inches and a half) when quite open, they form an exact foot rule ; and may be used as such in taking dimensions, being, like other common rules, divided into inches and parts.

The lines commonly laid down on the faces of this instrument to be used sector-wise, are as follow : *viz.* Upon one face, (that mark'd A in the print) 1. The lines of *Lines*, or of equal parts. 2. The lines of *Chords*. 3. The lines of *Secants*. 4. The lines of *Polygons*. On the other face, (mark'd B in the print) 1. The lines of *Sines*. 2. The lines of *Tangents* to 45 degrees. 3. Other lines of *Tangents* to a lesser radius, to supply the defect of the former, from 45 degrees to about 75 degrees.

Note, If the lines of the lesser tangents, and the lines of secants, be projected from a radius $\frac{1}{4}$ of the larger lines, and laid down upon the instrument to the same angle, (as is the practice of Mr. *Sisson*) then, without any alteration in the opening of the legs, any extent taken on the lines of lesser tangents or secants, 4 times repeated, will give the proper tangent or secant to the radius of the larger lines.

These several lines are projected and divided as the same lines upon the plain scale, but laid down upon the sector, (all except the lines of polygons) so as that they may all make one certain angle; that is, to whatever degree the sector is opened, the distance will be the same betwixt 100 and 100 on the lines of lines, 90 and 90 on the lines of sines, 60 and 60 on the lines of chords, and 45 and 45 on the lines of tangents. The lines of lines are each divided into 100 equal parts, each of which are again subdivided into halves and quarters, if the legs are of sufficient length; and with the lines of sines, chords and tangents, are each extended to the length of the radius of the circle from which they are projected; that is, the lines of sines to 90 degrees, the lines of chords to 60 degrees, and the lines of tangents to 45 degrees; each being numbered at every 10 degrees from the centre to their several ends, where they are mark'd with the initial letters of their respective names.

Note,

Note, The correspondent parallels of the several lines are intended for the better distinguishing the several divisions upon each.

The lines of secants, and of lesser tangents, are each projected from a circle, whose radius is $\frac{1}{4}$ of that from which the other lines are projected; and beginning at the length of that radius from the centre of the instrument, run to about 75 degrees. The tangents beginning at 45, where the larger lines of tangents end, have their degree distinguished by the numbers 45, 50, 60, 70, 75, and the secants with the numbers 10, 20, 30, &c. to 75; and both are mark'd, for distinction's sake, with the small initial letters of their names.

The lines of polygons are laid down from the centre of the joint, very near the inward edges of the instrument, and are numbered backwards, from the ends towards the centre, with the figures 4, 5, 6, &c. to 12, which comes within about three inches of the centre.

These are all the lines, which are usually laid down on this instrument to be used sector-wise.

The other lines, which are placed upon, and parallel to the outward edges on both faces, and used as on common scales, the sector being quite opened, are, on the edge itself, a line of *Inches*, with each inch subdivided into 10 equal parts;
and

and on the faces, the line of *Numbers*, commonly called *Gunter's-line*, with the lines of *Artificial Sines* and *Tangents*; also sometimes a line of *Hours* with a line of *Chords*, and a line of *Latitudes*, with a line of *Inclination of Meridians*, each being distinguished with its proper letter.

Of the Foundation and general Use of the Sector.

The foundation and general use of the sector depends upon the 4th proposition of the 6th book of *Euclid*; where it is proved, that the homologous sides of similar and equiangular triangles, are proportional to each other. Sup-

Fig. 7. pose therefore the lines AB , AC , to present any pair of lines upon the sector, particularly the lines of lines, making an angle BAC with each other. If then the line BC be drawn, and Aa , Ab taken equal, and the line ab also drawn; as Aa , or Ab is to AB or AC , so will ab be to BC . Whence, if the lines Aa , Ab , be the half, the third, the fourth, &c. of the lines AB , AC ; the lines ab will accordingly be the half, the third, the fourth, &c. of the line BC . As suppose AB or AC be 100, and Aa or Ab 50; then if BC be 60, ab shall be 30. Hence it follows, that if the lines AB , AC , represent the lines of chords, sines or tangents, that is, if the line AB or AC be the radius, and the line

line A a or A b, the chord, sine or tangent of some proposed number of degrees to the said radius A B; then the line a b will be the chord, sine or tangent of the same number of degrees to the radius B C.

And here note, that lines taken upon the sector are distinguished by the terms *lateral* and *parallel*; those being called lateral, which are found upon the sides of the instrument, as A B, A C; and parallel are such extents as are taken from one leg to the other, at equal distances from the centre of the joint, as B C, a b.

If therefore it be required to find the chord, sine or tangent of 20 degrees, to a radius of 5 inches; open the sector so that the distance from 60 to 60 on the line of chords may be equal to 5 inches; then will the parallel distance between 20 and 20, on the line of chords, be the chord of 20 degrees, to a radius of 5 inches; so likewise the distance between 20 and 20 on the lines of sines, and between 20 and 20 on the lines of tangents, will be the respective sine or tangent of 20 degrees, to the said radius of 5 inches.

If a chord of more than 60 degrees be required to any assign'd radius, as suppose the chord of 80 degrees; open the sector, as before directed, to the radius given, and with the said radius describe

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24 *The Description and Use of*

an arch sufficiently large, as AD; then
Fig. 8. taking the chord of half the number of
 degrees, viz. 40. lay it off twice upon
 the said arch, viz. from A to B, and from B to
 C; so shall a line drawn from A to C be the re-
 quired chord of 80 degrees.

If a tangent of more than 45 degrees be re-
 quired, as suppose the tangent of 70 degrees, it
 must be taken from the lines of lesser tangents;
 which if not laid down to the same angle as the
 larger lines, as in most sectors they are not, then
 must the sector be so opened, that the distance
 between 45 and 45, on the said lesser tangents,
 may be equal to the given radius; so shall the
 distance between 70 and 70 upon the said lines of
 lesser tangents, be the tangent of 70 degrees, to
 the radius given. But if the lines of lesser tan-
 gents are made (as they should be) to $\frac{1}{4}$ of the
 radius of the larger, and laid down to the same
 angle; then the tangent of any required number
 of degrees, either more or less than 45, may be
 taken off, without any alteration in the opening of
 the sector; for in the instance above of 70 degrees,
 the distance between 70 and 70 on the line of les-
 ser tangents, 4 times repeated, will be the tangent
 of 70 degrees, to the radius so set off between 45
 and 45 on the line of larger tangents.

To find the secant of any proposed number of
 degrees to a given radius, as suppose the secant
 of

of 60 degrees, to a radius of two inches; open the sector to the distance of 2 inches between 0 and 0, on the lines of secants; so shall the distance between 60 and 60, upon the said lines, be the secant of 60 degrees, to the radius of 2 inches: Or if, according to the above remark on the lesser tangents, the lines of secants are projected to $\frac{1}{4}$ of the larger radius, and form the common angle; the distance between any proposed point, 4 times repeated, will give the proper secant to the radius the larger lines are opened to.

If the contrary of any of these operations be required, that is, if a given right line be the chord, sine, tangent or secant of any certain number of degrees, as suppose 20, and the radius be sought; open the sector to the length of the given line, in the points 20 and 20 of the lines of chords, sines, tangents or secants, according as the said line is either chord, sine, tangent or secant; and the distance between 60 and 60 on the chords, between 90 and 90 on the sines, between 45 and 45 on the tangents, and between 0 and 0 on the secants, shall be the radius to the given line, whether it be chord, sine, tangent or secant.

This may suffice to shew the nature, general use, and excellence of this instrument; it evidently appearing, from what has been said, that the *Sector* is, as it were, an universal *Plain Scale*, to any radius not greater than the extent of the instru-

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ment itself, when quite open, or less than the distance between the numbers at the ends of the respective lines, when quite shut.

And as the several lines make equal angles with each other respectively, they are thereby adapted to the working of proportions in equal parts and sines, equal parts and tangents, &c. so that among other things, the several cases of Trigonometry may very readily be resolved by the sector. Some instances of which shall be hereafter given; but first I shall shew some particular uses of the lines of lines, and of the lines of polygons.

Of the particular Use of the Lines of Lines.

As lines of equal parts, their use is the same with such lines on common scales, serving to lay down any given length; but as they are laid down sector-wise, they have other various uses, some of the principal of which are shewn in the following problems.

P R O B L E M I.

To divide a given Line into any Number of equal Parts.

Take the extent of the given line between the feet of your compasses, and set off that extent as a parallel between the legs of your sector, in the points expressing the number of parts into which
the

the line is to be divided; so shall the parallel distance between 1 and 1 divide the given line into the parts required. For example, suppose the line A B were given to be divided into 6 equal parts; set off the extent of the line A B, betwixt the legs of the sector in the points 6 and 6, and the parallel distance betwixt 1 and 1, viz. A C, shall be the $\frac{1}{6}$ part of the line A B. *Fig. 9.*

If the given line be of too great extent to be applied to the legs of the sector, take one half, or one fourth of it, which being divided, as above directed, into the parts required, the double, or quadruple of one of those parts, will accordingly divide the whole line into the number of parts proposed.

PROBLEM II.

To divide a Line in the same manner as another is divided.

Lay down the divided line laterally on both sides of the sector, from the centre, and mark the point of its extension; then opening the sector to the extent of the second line, in the terms of the first; apply likewise the several parts of the first line laterally on the legs of the sector; so shall the several parallels, in the terms of those parts, be the correspondent parts in the line to be divided. Thus, if the line C D were to be divided in the manner as the line A B; *Fig. 10.*

placing AB on the lines of lines, in AB and AC open the sector to the extent of CD, in the terms BC; the sector remaining in this position, set off the parts AE and AF, of the line AB, on both legs of the sector, in AE and AF, so shall the parallel EE give the division CG, and the parallel FF, the division GH, on the line CD to be divided.

PROBLEM III.

To find the Proportion between two, or more given Lines.

The proportion between any given lines may be measured upon either of the lines of lines, as on other scales of equal parts, without opening the instrument. But to perform the operation sector-wise, open the sector, so that the extent between 100 and 100, may be equal to the greater of the given lines; then taking the lesser lines severally between your compasses, carry them along the legs of the sector, till both points rest exactly on the same number upon the lines of lines; which numbers will accordingly give their proportion to 100. Thus if it were required

Fig. 11. to find the proportion between the lines AB and CD; having set off the extent of the longer line AB, betwixt the points of 100 and 100, apply the shorter line CD parallel to the former, and you will find it cut the lines of lines, in the points 80 and 80. The proportion therefore

therefore of CD to AB is as 80 to 100; or in smaller numbers, as 4 to 5.

If the greater line AB be too long to be applied in the points 100 and 100, then supposing the lesser line CD 100, cut off AE upon the greater line, equal to CD, and the part AE being set off on the sector, as before directed, in the points 100 and 100, the part EB will be found to cut the lines of lines in the numbers 24 and 24, so that the proportion of the part EB, to the part AE, will be found to be as 24 to 100; therefore the proportion of the line CD, to the line AB, will this way be found to be as 100 to 124.

PROBLEM IV.

To increase or diminish a Line in any given Proportion.

Set the extent of the given line betwixt the legs of the sector, in the points of the number given, on each of the lines of lines; and the parallel distance, betwixt the points of the number required, will give the line sought. Thus, if the line A is to be increased in the proportion of 3 to 7; set off the length of Fig. 12. the line A, betwixt 3 and 3, on the lines of lines; so shall the parallel distance betwixt 7 and 7, be the increased line B sought. On the contrary, if the line B was to be diminished in the

the proportion of 7 to 3, set off the line B betwixt 7 and 7, and the parallel distance betwixt 3 and 3 shall be the diminished line A sought. Note, it may be sometimes more convenient to work with the equimultiples of the numbers given, which will produce the same effect; thus, for the proportion of 3 to 7, you may take the numbers 6 to 14, 9 to 27, 12 to 28, or 30 to 70, &c.

PROBLEM V.

To find a third Proportional to two given Lines.

Lay down both the given lines laterally on the legs of the sector, observing the points of their extension; then taking the second line between your compasses, open the sector to its extent in the terms of the first line; so shall the parallel distance, between the terms of the second line, be the third proportional sought. For example,

let the two given lines be A and B, which being laid down on the sides of the sector from the centre A, suppose the terms of the first line to be BB, and the terms of the second CC; then opening the sector in the terms BB, to the extent of the second line B, the parallel distance between CC shall be the third proportional sought. For as A is to B, so is BB, equal to the line B, to CC.

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PROBLEM VI.

To find a fourth Proportional to three Lines given.

Let the first and third lines be both laid down laterally on the legs of the sector, from the centre; then opening the sector in the terms of the first line, to the extent of the second line; the parallel distance between the terms of the third line shall be the fourth proportional.

For example, let A, B and C, be the *Fig. 14.* three lines given; set off A and C on the sides of the sector, from the centre A, marking the terms of their extension BB and CC; then taking the second line B, open the sector to its extent, in the terms of the first line A, *viz.* BB; and the parallel distance betwixt CC, the terms of the third line C, shall be the fourth proportional sought.

Note, In Geometry, as well as in Arithmetick, it is not necessary that all the proportionals be of one denomination, but only that the fourth found correspond with the second, as the third with the first; therefore the first and third proportionals may be measured by one scale, and the second and fourth by another: Or their respective proportions to each other may be found, as directed in *Prob. III.*

P R O B L E M VII.

To find a Right Line nearly equal to the circumference of a given Circle.

Since the proportion of the diameter of a circle to its circumference is nearly as 3. 18 to 10; therefore opening the sector to the length of the diameter of the circle proposed, in the points on the lines of lines, answering to the number 3. 18, the parallel distance between 10 and 10 will be the length of a line nearly equal to the circumference of the given circle.

P R O B L E M VIII.

To open the Sector so that the two Lines of Lines (and consequently the others respectively, if they are laid down as they ought, at equal angles) may stand at right angles with each other.

Since by the 47th proposition of the 1st book of *Euclid* it is demonstrated, that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of both the legs; therefore, if the whole lateral length of one of the lines of lines, taken as 10 equal parts, be set over from 8 on one leg to 6 on the other, the two lines of lines (and consequently, the rest respectively) shall stand at right angles with each other, For 64, the square of 8, added to 36, the square of 6, makes 100, equal to the square of 10.

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To these few problems, peculiar to the lines of lines, I shall add one or two of a more general nature.

P R O B L E M IX.

To open the Lines upon the Sector, or the Edges of the instrument itself, to any proposed angle.

Take the quantity of the angle proposed, laterally from the centre, on one of the lines of chords, and set it off as a parallel betwixt 60 and 60 on the said lines; so shall the two lines of chords, and every other pair of lines, stand respectively with each other at the determined angle. Thus, to open the lines to an angle of 20 degrees, set off the lateral extent of 20 degrees taken from the centre, upon either of the lines of chords, as a parallel betwixt 60 and 60 upon the said lines; and the two lines of chords, and consequently all the rest, each being laid down to the same angle, shall accordingly form an angle of 20 degrees, each with the other respectively. And if (as in the print prefix'd to this work) every pair of lines form with each other an angle of 6 degrees, when the sector is quite shut, it is but adding 6 degrees to the quantity of the angle proposed, in taking the lateral extent, and the parallel so set off shall set the edges of the instrument itself to the angle required.

P R O B L E M X.

The Sector being opened, to find the Quantity of the Angle, which either the several Lines, or the Edges of the Instrument, make with each other.

This problem is but the reverse of the former : For if the parallel distance betwixt 60 and 60 on the line of chords, be applied laterally from the centre, on either of the said lines, the degrees of its extension are the measure of the angle the several lines respectively make with each other ; deducting therefore 6 degrees from the quantity so found, for the angle the several lines make with each other, when the instrument is quite shut ; and the remainder will be the quantity of the angle formed by the edges of the instrument.

Of the Use of the Lines of Polygons.

The lines of polygons are chiefly contrived for the ready division of the circumference of a circle into any number of equal parts from 4 to 12, that is, as a ready means to inscribe regular polygons of any given number of sides, from 4 to 12, within a given circle : To do which, set off the radius of the given circle, (which is always the side of the inscribed hexagon) as a parallel betwixt 6 and 6, upon the said lines ; so shall the parallel distance betwixt 5 and 5, be the side of a pentagon ; between 7 and 7, the side of a heptagon

tagon; between 8 and 8, the side of an octagon, &c. All which is too plain, even to need an example.

N. B. All such regular polygons, whose number of sides will exactly divide 360, (the number of degrees into which all circles are supposed to be divided) without a remainder, may likewise be very readily set off upon the circumference of a circle, from the lines of chords; for laying down the radius of the circle, as a parallel betwixt 60 and 60, upon the lines of chords; the sector remaining so opened, having divided 360 by the number of sides, the parallel distance betwixt the numbers of the quotient shall be the side of the polygon required; thus, for an octagon, take the extent betwixt 45 and 45; and for a pentedecagon, the extent betwixt 24 and 24, &c.

If it be required to form a polygon, upon a given right line; set off the extent of the given line, as a parallel betwixt the points, upon the lines of polygons, answering to the number of sides, of which the polygon is to consist; as for a pentagon, betwixt 5 and 5; or for an octagon, betwixt 8 and 8; so shall the parallel distance betwixt 6 and 6 be the radius of a circle, whose circumference shall be divided, by the given line, into the number of sides required.

The lines of polygons are likewise useful in describing, upon any given line, an isosceles triangle, whose base angles shall each be double that at the vertex. For, taking the given line betwixt your compasses, open the sector, till that extent reaches from 10 on one leg to 10 on the other; then shall the parallel distance betwixt 6 and 6 be the length of the two equal sides of the triangle sought.

Before we proceed to the manner of working proportions upon the sector, it will be necessary to consider the nature of the lines of artificial numbers, sines and tangents; because as the same proportions may likewise be work'd upon these artificial lines, so one method of working will serve as a proof of the other.

Of the Lines of Artificial Numbers, Sines and Tangents.

These several lines, which are usually laid down upon sectors parallel to the outward edges, in the same manner as upon other scales, are only the logarithms of the natural numbers, sines, and tangents, thus transferred to a scale.

1. The line of numbers, or *Gunter's line*, as it is commonly called, is a line of geometrical proportion, divided into 9 unequal parts, beginning

ning at 1 towards the left hand, and numbered on with 2, 3, 4, 5, &c. to 10, about the middle of the line; where another radius begins, and the same divisions are repeated, numbered as before to 10, at the end of the line on the right hand. Each of these primes, or first grand divisions, are subdivided, according to the same ratio, into 10 other parts, and each of these subdivisions (if the line be of sufficient length) should again be subdivided into 10 other lesser parts. But upon pocket-sectors, which, when opened, are but 12 inches in length, only the part from 1 in the middle, to 2 towards the right hand, is a second time divided, and that but into 5 parts instead of 10, every one of which must therefore be accounted as 2 centesms. In numbering therefore upon the line, (wherein consists the greatest difficulty) the figures 1, 2, 3, &c. which denote the primes, may be taken arbitrarily, either as units, tens, hundreds, or thousands; or they may represent tenth, hundredth, or thousandth parts of an unit.

If 1, at the beginning of the line, be taken for unity, then 1 in the middle will be 10, and 10 at the end 100; but if the first 1 be accounted 10, then 1 in the middle will stand for 100, and 10 at the end for 1000; again, if the first 1 be reckoned 100, then 1 in the middle will be 1000, and 10 at the end 10000.

When

When therefore the first 1, or prime, represents 10 units, the figures 2, 3, 4, &c. to 1 in the middle, will stand for 20, 30, 40, &c. and each tenth, or subdivision in the first radius of the line, will signify 1 unit, and each centesim in those tenths (if there be any) will be $\frac{1}{10}$ part of an unit; 1 in the middle will be (as before observed) 100, and the figures 2, 3, 4, &c. following 200, 300, 400, &c. each tenth, or subdivision in the second radius, will denote 10 units, and each centesim 1 unit. Again, let the first 1 represent 100, then the figures 2, 3, 4, &c. will signify 200, 300, 400, &c. 1 in the middle 1000, and 10 at the end 10000; according to this supposition, in the first radius, each tenth of a prime will be 10 units, and each centesim 1 unit; and in the second radius, each tenth of a prime 100, and each centesim 10.

In accounting decimal fractions upon the line; if 1 in the middle of the line be accounted as unity, then will each prime in the first radius denote 1, each tenth of a prime .01, and each centesim .001. If ten at the end of the line to the right hand be accounted as unity, the whole first radius will represent 1, and the numbers 2, 3, 4, &c. in the second radius 2, 3, 4, &c. every prime in the first radius, and every tenth of a prime in the second radius, will signify .01, or one hundredth part of unity; and every tenth of a prime in the first radius, or centesim in the second, will denote .001, or one thousandth part of unity.

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The numeration of the line thus explained, let it now be proposed to find the point thereon, answering to the number 436. For the 4 hundred, take the 4 first primes next the left hand; for the second figure 3, take 3 tenths of the grand division betwixt 4 and 5, and for the 6 units, reckon 6 centesms, or 6 parts of the next tenth; so that the extent from 1 at the beginning of the line, to that point, will express the number 436. The same point will likewise represent the numbers 43, 6. 4, 36. or ,436; if the first 1 be accordingly accounted 10, 1, or $\frac{1}{10}$ of unity.

To multiply upon the line, extend your compasses from 1 at the beginning, to the point representing the number of your multiplier; the same extent will reach from the number of your multiplicand, to the product; thus, if 125 were given to be multiplied by 42, extend the compasses from 1 to 42, and the same extent will reach from 125 to 5250.

To divide upon the line, extend the compasses backwards from the number of the divisor, to unity; the same extent, laid the same way, will reach from the number of the dividend to the quotient: Thus, if 5250 were given to be divided by 125, extend the compasses backwards from 125 to unity, the same extent will reach the same way, from 5250 to 42.

2. The line of artificial lines consists only of the logarithms of the natural lines, transferred from a table of logarithms to the scale; they are numbered from the left to the right, with the figures 1, 2, 3, &c. to 10, which stands about the middle of the line; and so forward with 20, 30, 40, &c. to 90, at the end on the right. In the first part of the line, the grand divisions each represent one degree; so that if each prime be subdivided into 12 parts, (as they commonly are upon sectors) each subdivision will signify 5 minutes. In the latter part of the line, the grand divisions, which are each 10 degrees, being subdivided into 10 parts, each subdivision will represent 1 degree; and according as they are again subdivided into 4, 3 or 2 parts, such second subdivisions contain 15, 20, or 30 minutes each.

3. What has been said of the line of sines is likewise to be understood of the line of artificial tangents, whose divisions begin also at 1, and run on to 10 in the middle of the line, signifying single degrees; in the second part it runs on with 20, 30, &c. to 45, which stands at the end of the line; then returns back again to 90, where you begun, as is evident from the inspection of the lines themselves.

The use of these lines is in working proportions, whether arithmetical or geometrical; particularly they are of very ready service in trigonometrical solutions.

The

The method of working with them is thus: In all proportions you have three terms given, to find a fourth. Seek out therefore the first term, whether number, sine, or tangent, on its proper line; and in that point set one foot of your compasses, extending the other to the second or third term, which-ever of them is of the same name with the first; the same extent laid from the other term, the same way, will reach to the fourth term required.

Thus, to work the following proportion; as the sine of $52^{\circ} 30'$ to the number 85, so is radius, or the sine of 90° , to the fourth number required. Set one foot of your compasses in the line of artificial sines on $52^{\circ} 30'$, and extend the other foot to 90° on that line, the same extent will reach from 85 on the line of numbers to 107, the fourth number required. And thus are all questions in proportion worked by the artificial lines of numbers, sines and tangents; as will more fully be seen in the following section.

Of the manner of working Proportions on the Sector, both by the conjoint Use of the Lines of Lines, with the Lines of natural Sines, Tangents, &c. as also by the artificial Lines of Numbers, Sines and Tangents.

Having already, in treating of the reason and general use of this instrument, shewn the particular use of the several lines of chords, sines, tangents, and secants laid down thereon, and how universal their application is thereby made, above what it is upon the plain scale; I now proceed to shew the conjoint use of the lines of lines, with the lines of natural sines and tangents, as they are laid down sector-wise; and also, the use of the artificial lines of numbers, sines, and tangents, in working proportions; which I shall exemplify in the resolution of the several cases of plain Trigonometry, both right-angled, and oblique-angled.

The Seven Cases of Right-angled Plain Triangles.

C A S E I.

The angles, and one of the legs being given, to find the other leg.

As, suppose in the triangle ABC,
Fig. 15. the angle A is 36 degrees, and the base
 AB

AB 75 feet, it were required to find the height of the perpendicular BC.

The trigonometrical proportion for working this case is, as radius to the tangent of the angle A, so is the leg AB to the leg BC.

To work which upon the sectoral lines,

Take with your compasses, on either of the lines of tangents, the tangent of 36 degrees laterally from the centre, and set off that extent parallel-wise from 45 to 45, on the lines of tangents; with this opening of the sector, take between the feet of your compasses the parallel extent betwixt 75 and 75, the length of the given leg, on the lines of lines; which extent, applied laterally on either of the lines of lines, will reach to 54, for the length of the other leg; so that the perpendicular BC is thereby found to be 54 feet.

Upon the artificial lines,

Set one foot of your compasses in 45, (the radius) upon the line of tangents, and extend the other backwards to 36; the same extent will reach upon the line of numbers, from 75, the length of the given leg AB, to 54, the length of the required leg BC.

C A S E II.

The angles, and one of the legs being given, to find the Hypothenufe.

As, suppose in the former triangle, the base AB, as before, 75 feet, the angle A 36 degrees, and consequently the angle C 54 degrees; and it were required to find the length of the hypothenufe AC.

Proportion. As the sine of the angle C, to the radius, so is the base to the hypothenufe.

To work which upon the sectoral lines,

Take, with your compasses on either of the lines of sines, the sine of 54 degrees (the quantity of the angle C) laterally from the centre, and set off that extent, as a parallel betwixt 90 and 90, on the lines of sines; the sector remaining in that position, take between the feet of your compasses the lateral distance from the centre to 75, (the length of the base) on either of the lines of lines; and that extent applied parallel-wise, will reach from 93 to 93 upon the lines of lines, the length of the hypothenufe sought.

Upon the artificial lines,

Set one foot of your compasses in 54, (the sine of the angle C) upon the line of sines, and extend the

the other to 90 (the radius); the same extent will reach upon the line of numbers, from 75, the length of the base, to 93, the length of the hypotenuse required.

C A S E III.

The hypotenuse and angles being given, to find either of the legs.

As, suppose in the foregoing triangle, the hypotenuse AC 93, the angle A 36. degrees, and consequently the angle C 54 degrees; and it were required to find the length of the base AB.

Proportion. As radius. to the sine of the angle C, so is the hypotenuse to the base.

To work which upon the sectoral lines,

Setting one foot of your compasses on the centre of one of the lines of sines, extend the other laterally to 54 degrees, and set off that extent as a parallel betwixt 90 and 90 on the said lines of sines; the sector continuing so opened, take the parallel distance between 93 and 93, the length of the hypotenuse, on the line of lines; and applying it laterally from the centre, upon one of the said lines, the extent will be found 75, for the length of the base required.

Upon

Upon the artificial lines,

Set one foot of your compasses in 90, on the line of fines, and extend the other backwards to 54, the sine of the angle C; the same extent will reach, upon the line of numbers, from 93, the length of the hypotenuse, backwards to 75, the length of the base required.

C A S E IV.

The legs being given, to find the angles.

As, suppose in the foregoing triangle, the base AB, as before, 75 feet, and the perpendicular BC 54 feet, and it were required to find the quantities of the angles A and C.

Proportion. As the leg AB to the leg BC, so is radius to the tangent of the angle at A; whose complement to a quadrant will be the angle at C.

To work which upon the sectoral lines,

Setting one foot of your compasses on the centre, extend the other laterally on one of the lines of lines to 54, the length of the perpendicular BC, and set off that extent as a parallel between 75 and 75, the length of the base, upon the said lines of lines; the sector continuing thus opened, take

take the parallel radius, or extent between 45 and 45 on the lines of the tangents, between your compasses, which laid down laterally from the centre, on either of the lines of tangents, will give 36° for the quantity of the angle A; and that, subtracted from 90, will leave 54° , for the quantity of the angle C.

Upon the artificial lines,

Set one foot of your compasses upon the line of numbers, in 75, the length of the base; and extend the other backwards to 54, the length of the perpendicular; the same extent will reach from 45 (radius) on the tangents, to 36 the tangent of the angle A.

C A S E V.

The legs given, to find the hypotenuse.

As, suppose in the foregoing triangle, the base AB is 75, and the perpendicular BC 54; it were required to find the length of the hypotenuse AC.

To work this case upon the sectoral lines,

Open the instrument to a right angle (by *Prob.* the last, of the uses of the lines of lines); then taking between your compasses the slant extent from 75, on one of the said lines of lines,
to

to 54 on the other, apply the said extent laterally from the centre, on either of the said lines, and you will find 93 for the length of the hypotenuse required.

Upon the artificial lines,

First find the angles, by *Case* the 4th; then the hypotenuse, by *Case* the 2d.

C A S E VI.

The hypotenuse and one of the legs given, to find the angles.

As, suppose in the foregoing triangle, the hypotenuse AC 93, and the base AB 75; and it were required to find the angles A and C.

Proportion. As the hypotenuse to the base, so is radius to the sine of the angle at C.

To work which upon the sectoral lines,

Setting one foot of your compasses in the centre, extend the other laterally on one of the lines of lines to 93, the length of the hypotenuse AC; this extent set off parallel-wise, as a radius betwixt 10 and 10, on the said lines of lines; then from the centre laterally on one of the lines of lines, taking the extent of the base AB 75 between your compasses, run that distance parallel-wise

wise along the lines of fines, till the feet rest in like numbers, which will in this example be at 54 degrees, the quantity of the angle C, opposite to the base; which being subtracted from 90, there will remain 36 degrees, the quantity of the angle A.

Upon the artificial lines,

Set one foot of your compasses upon the line of numbers, in 93, the length of the hypotenuse, and extend the other backwards to 75, the length of the base; the same extent will reach upon the line of fines, from 90 (radius) backwards to 54, the sine of the angle C; which, subtracted from 90, leaves 36 for the angle A.

C A S E VII.

The hypotenuse, and one of the legs, being given, to find the other leg.

As, suppose (as before) in the foregoing triangle, the hypotenuse AC 93, and the base AB 75, and it were required to find the perpendicular BC.

To work this Case upon the sectoral lines,

Open the sector to a right angle (by *Prob.* the last, of the uses of the lines of lines); then taking between your compasses the length of the hypotenuse AC 23, from the centre, upon either of
H the

the lines of lines, set one foot in 75, the length of the base AB upon one line; and the other foot, being turned towards the centre, will rest upon the other line in the point 54, the length of the perpendicular BC required.

Upon the artificial lines,

First find the angles, (by *Case* the sixth) then to find the length of the perpendicular, set one foot of your compasses on the line of sines in 90, (radius) and extend the other backwards to 36, the sine of the angle A; the same extent will reach, on the line of numbers, from 93, the length of the hypotenuse, backwards to 54, the length of the perpendicular required.

The Six Cases of oblique-angled plain Triangles.

CASE I.

Two sides, and an angle opposite to one of them, being given, to find the other opposite angle.

As, suppose in the triangle ABC, *Fig. 16.* the side AC being 65 leagues, and the side BC 54 leagues, and the angle A opposite to the side BC $35^{\circ} 30'$, it were required to find the quantity of the angle B.

Propor-

Proportion. As the side opposite to the given angle, to the sine of the given angle; so is the side opposite to the angle sought, to the sine of the angle required.

To work which upon the sectoral lines,

Setting one foot of your compasses on the centre, extend the other laterally on the lines of lines to 54, the length of the side BC; which extent set off as a parallel betwixt $35^{\circ} 30'$, and $35^{\circ} 30'$ on the lines of sines, the quantity of the given angle A; then from the centre of the lines of lines, take the length of the side AC 65 between your compasses, and run that distance parallel-wise along the lines of sines, till the feet rest in like numbers, which in this example will be at $44^{\circ} 15'$ nearly; and that shall be the quantity of the angle B, opposite to the side AC.

If you would know the quantity of the remaining angle C, subtract the sum of the angles A and B from 180 degrees (the sum of the three angles of every triangle); the remainder $100^{\circ} 15'$, shall be the quantity of the angle C.

Upon the artificial lines,

Set one foot of your compasses on the line of numbers in 54, the length of the side BC, opposite to the given angle A, and extend the other to 65, the length of the side AC; the same extent.

will reach upon the line of fines, from $35^{\circ} 30'$, the given angle A, to $44^{\circ} 15'$, the quantity of the required angle B.

C A S E II.

The angles, and one side being given, to find either of the other sides.

As, suppose in the foregoing triangle, the side BC, as before, being 54 leagues, the angle A $35^{\circ} 30'$, the angle B $44^{\circ} 15'$, and the angle C $100^{\circ} 15'$, the sides AB and AC were required.

Proportion. As the sine of the angle opposite to the given side, to the side given; so is the sine of the angle opposite to the required side, to the side required.

N. B. When an obtuse angle is given, you must work by its complement to 180 degrees.

To work this Case upon the sectoral lines,

From the centre of the lines of lines, take the length of the given side BC 54 laterally between your compasses; which extent set off as a parallel betwixt $35^{\circ} 30'$, and $35^{\circ} 30'$, the sine of its opposite angle A, upon the lines of fines: the sector thus opened, take between your compasses the parallel distance between $44^{\circ} 15'$, and $44^{\circ} 15'$, the quantity of the angle B, upon the lines of fines; and

and applying the said distance laterally upon either of the lines of lines, it will give 65 for the length of the opposite side AC. So likewise the parallel distance between $79^{\circ} 45'$, and $79^{\circ} 45'$, (the complement of $100^{\circ} 15'$, the quantity of the angle C) will, when applied laterally on either of the lines of lines, give 92 for the length of the side AB.

Upon the artificial lines,

Set one foot of your compasses on the lines of sines in $35^{\circ} 30'$, the quantity of the angle A, opposite to the given side BC, and extend the other to $44^{\circ} 15'$, the quantity of the angle B; the same extent will reach upon the line of numbers from 54, the length of the given side BC, to 65, the length of the required side AC. Or setting the first foot as before in $35^{\circ} 30'$ on the sines, extend the other to $79^{\circ} 45'$ (the complement of $100^{\circ} 15'$ to 180°); the same extent will reach on the line of numbers, from 54 to 92, the length of the opposite side AB.

C A S E III.

Two sides, and an angle opposite to one of them being given, to find the other side.

As, suppose in the foregoing triangle, the side AC being 65 leagues, and the side BC 54 leagues,
and

and the angle A, opposite to the side BC, $35^{\circ} 30'$, it were required to find the side AB.

To work this Case either upon the sectoral or artificial lines.

1. Find the angles, by Case the first.

2. The side, by Case the second.

CASES IV. and V.

Two sides and the contained angle being given, to find the other two opposite angles, and the other side.

As, suppose in the foregoing triangle, the side AC, as before, 65 leagues, and the side AB 92 leagues, and the contained angle A $35^{\circ} 30'$, it were required to find the other opposite angles B and C, with the side BC.

The proportion for finding the angles in this case is as follows: *viz.* As the sum of the two sides AC and AB, to their difference, so is the tangent of half the sum of the angles B and C, *viz.* $72^{\circ} 15'$, to the tangent of half their difference; which, added to their half sum, gives the greater angle C; but subtracted from the said half sum, leaves the lesser angle B.

N. B,

N. B. The sum of the angles B and C is found by subtracting the given angle A, viz. $35^{\circ} 30'$, from 180° .

To work these Cases upon the sectoral lines,

These two cases are taken together, because upon the sectoral lines it is necessary to find the length of the third side, before the angles can be discovered. To find therefore the length of the side BC, take the quantity of the given angle A $35^{\circ} 30'$, laterally upon one of the lines of sines, and set it off as a parallel betwixt 90 and 90, on the said lines: The sector thus opened, set one foot of your compasses in 92, the extent of the side AB, upon one of the lines of lines; and extend the other to 65, the length of the other given side AC, upon the other line of lines; this extent measured laterally from the centre, upon either of the lines of lines, will give 54 for the length of the required side BC: Then may the quantity of either of the required angles be found by Case the first; which, added to the given angle, and the sum subtracted from 180, will give the quantity of the other angle.

Upon the artificial lines,

First, to find the angles: Set one foot of your compasses upon the line of numbers in 157, (the sum of the sides AB and AC) and extend the other backwards to 27 (the difference of the sides);

sides); the same extent will reach on the line of tangents from $72^{\circ} 15'$ (half the sum of the angles B and C) to 28° ; which, added to $72^{\circ} 15'$, gives $100^{\circ} 15'$ for the greater angle C; or, subtracted from $72^{\circ} 15'$, leaves $44^{\circ} 15'$, the lesser angle B. The angles thus found, the side BC may be obtained by *Case 2*.

C A S E VI.

The three sides being given, to find the angles.

As, suppose in the foregoing triangle, the side AB 92 leagues, the side AC 65 leagues, and the side BC 54 leagues.

To work this Case upon the sectoral lines,

First, fixing upon any one angle, as suppose the angle A, note the extent of the two sides which include it, viz. AB 92, and AC 65; then taking the extent of the third side BC 54 between your compasses, set one foot in 92, upon one of the lines of lines, and open the sector, till the other foot falls upon 65, on the other line of lines: The sector thus opened, take betwixt your compasses either the parallel distance between 30 and 30 in the sines; which measured laterally from the centre on either of the lines of sines, will give $17^{\circ} 45'$, half the quantity of the angle sought: or take the parallel radius, viz. the extent betwixt 60 and

and 60 on the chords; which, measured laterally on either of the lines of chords, will give you the whole quantity of the required angle A, *viz.* $35^{\circ} 30'$. The angle B may either be found by the same method, from the three given sides, or by the help of the discover'd angle A, according to the method laid down in *Case 1*.

The two angles A and B thus found, subtract their sum from 180° , the remainder will be the quantity of the third angle C.

Upon the artificial lines,

To work this case upon the artificial lines, first divide the base, or longest side AB, into two segments, by a perpendicular let fall from the opposite angle C; the value of which segments are discoverable by the following proportion, *viz.* As the true base to the sum of the other two sides, so is the difference of the said two sides to the difference of the segments of the base; which difference, added to the true base, will give the double of the greater segment; but subtracted, will leave the double of the lesser segment.

Set therefore one foot of your compasses on the line of numbers in 92, the length of the true base, and extend the other to 119, the sum of the two other sides; the same extent will reach on the same line, from 11, the difference of the sides, to

14, 228, the difference of the segments of the base; which difference 14, 228, added to the true base 92, gives 106, 228, the double of the greater segment; but subtracted, leaves 77, 772, the double of the lesser segment.

The triangle therefore being divided into two rectangled triangles, by the perpendicular, and the segments of the base thus found, the angles may be obtained by *Case* the 6th of rectangled plain triangles.

I have now, I presume, pretty well exemplified the uses of the several lines laid down upon the modern sectors; tho', besides what I have mentioned, they are capable of very various applications, in mensuration, architecture, dialling, astronomy, &c. But as some sectors, particularly the *French* ones, are furnished with lines of *Superficies*, and *Solids*; before I conclude, I will just give the manner of their projection, and application.

Projection of the lines of superficies.

Let the lines AE AE represent the lines of superficies; to divide which, having a scale of 1000 equal parts of the same length as the intended lines, take therefrom *Fig. 17.* 125 parts; which set off from the centre A, as the first division upon each; 250 parts will give you the fourth division; the mean proportion between 125 and 150, will be the number of parts marking the second division; 375 parts taken from the scale, and set off from the centre A, will reach to the ninth division; the third division will be the mean proportion between 125 and 375. Again, 625 parts will give the 25th division; and the mean proportion between 125 and 625, will be the fifth division, &c. lastly, 1000 parts will be the 64th division: All which is plain from the consideration, that similar surfaces are in a duplicate ratio of their homologous sides.

Use of the line of superficies.

Taking between the points of your compasses the radius of a circle, or the side of any plain figure, and setting it as a parallel betwixt 1 and 1, the first division from the centre; so shall the

1 2

parallel

parallel extent betwixt 2 and 2, give you the radius of a circle, or side of a figure of a double area; and betwixt 3 and 3 triple, &c.

In the same manner may be found circles, and other plain figures sub-double, sub-triple, &c.

Projection of the lines of solids.

Let the lines A F A F represent the lines of solids: From the said scale of 1000 equal parts, set off from the centre A 250, for the first division, and 500 for the 8th; then finding two mean proportionals betwixt 250 and 500, the first will give you the second division; and the second the fourth: To find this first mean proportional, subtract the logarithm of 250 from the logarithm of 500, and adding the $\frac{1}{3}$ of what remains to the logarithm of 250, you will get the logarithm of the first mean proportional; which will be nearly 315: and if you add again the same $\frac{1}{3}$ to the logarithm of 315, you will have the second mean proportional. For the third division, take the first of two mean proportionals between 250, and the triple thereof, 750; and for the fifth division, take the first of two mean proportionals between 250 and 1250, five times 250, &c. Lastly, the whole line A G, or a H, being 1000 equal parts, will be the 64th division.

Use of the line of solids.

Set over the side of any body, as a parallel betwixt 1 and 1, the first division on the lines of solids; and the parallel extent betwixt 2 and 2, shall give the homologous side of a similar body, whose solid content shall be double; and so the several other divisions will accordingly give sides of bodies triple, quadruple, &c. or sub-double, sub-triple, sub-quadruple, &c.

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